

# Summer Review Packet

## For Students Entering AP Calculus Courses

### In the 2009-2010 School Year

So you don't start thinking your Math teachers care less about you than your History or English teachers, we have prepared this brief review of topics you have previously studied. Your completion of the problems in this packet prior to your return to school in September will benefit you as you gear up for your Advanced Placement Calculus course. **Complete all work in the packet and have it ready to be turned in to your Calculus teacher on the first day of classes in September. Since the answers are included on the last pages of this packet, the work will be what is graded. SO SHOW ALL WORK! A calculator is not to be used while working on this packet.**

Certainly, you may complete this assignment at any time during the summer, but it will most likely benefit you the most if you find time in the few weeks prior to the start of the school year. Of course, if you are at the beach, on the ocean, in the mountains or a foreign country, your mind may wander while completing the assignment, but that will not be considered as a bad thing. For extra credit, please describe the most exotic location where you did any work on the packet. The *range* of answers here will prove to be interesting reading, we're sure. (Do not include sand between the pages as evidence though!) Your *composition* of work will be considered ultimately as a *function* of how much effort you *log*, naturally. Stay *rational* about it, by all means – we are only asking for a small *fraction* of your time! It would be *improper* to see this as anything more than a brief review. Be sure to *sine* the top of the first page before handing in your work.

Enjoy your summer,  
The Math Department

I. Simplify.

1.  $\frac{x-4}{x^2-3x-4}$

2.  $\frac{x^3-8}{x-2}$

3.  $\frac{5-x}{x^2-25}$

4.  $\frac{x^2-4x-32}{x^2-16}$

II. Fill in.

1. The 3 Pythagorean Identities: \_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_

2.  $\cos(2x) =$  \_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_

3.  $\sin(2x) =$  \_\_\_\_\_

III. Simplify.

1.  $\frac{1}{x+h} - \frac{1}{x}$

2.  $\frac{2}{\frac{x^2}{10} - x^5}$

3.  $\frac{\frac{1}{3+x} - \frac{1}{3}}{x}$

4.  $\frac{2x}{x^2-6x+9} - \frac{1}{x+1} - \frac{8}{x^2-2x-3}$

IV. Solve for z.

1.  $4x + 10yz = 0$

2.  $y^2 + 3yz - 8z - 4x = 0$

V. If  $f(x) = \{(3,5), (2,4), (1,7)\}$ ,  $g(x) = \sqrt{x-3}$ ,  $h(x) = \{(3,2), (4,3), (1,6)\}$  and  $k(x) = x^2 + 5$ , find:

1.  $(f+h)(1)$

2.  $(k-g)(5)$

3.  $(f \circ h)(3)$

4.  $(g \circ k)(7)$

5.  $f^{-1}(x)$

6.  $k^{-1}(x)$

7.  $\frac{1}{f(x)}$

8.  $(kg)(x)$

VI. Follow the directions for each problem.

1. Evaluate  $\frac{f(x+h) - f(x)}{h}$  and simplify if  $f(x) = x^2 - 2x$ .

2. Expand  $(x+y)^3$

3. Simplify  $x^{\frac{3}{2}} \left( x + x^{\frac{5}{2}} - x^2 \right)$

VII. Expand and simplify.

1.  $\sum_{n=0}^4 \frac{n^2}{2}$

2.  $\sum_{n=1}^3 \frac{1}{n^3}$

VIII. Simplify.

1.  $\frac{\sqrt{x}}{x}$

2.  $e^{\ln 3}$

3.  $e^{(1+\ln x)}$

4.  $\ln 1$

5.  $\ln e^7$

6.  $\log_3\left(\frac{1}{3}\right)$

7.  $\log_{1/2} 8$

8.  $\ln \frac{1}{2}$

9.  $e^{3\ln x}$

10.  $\frac{4xy^{-2}}{12x^{-1/3}y^{-5}}$

11.  $27^{2/3}$

12.  $(5a^{2/3})(4a^{3/2})$

13.  $(4a^{5/3})^{3/2}$

14.  $\frac{3(n+1)!}{5n!}$

IX. Using the point-slope form  $y - y_1 = m(x - x_1)$ , write an equation for the line:

1. with slope  $-2$ , containing the point  $(3,4)$

2. containing the points  $(1,-3)$  and  $(-5,2)$

3. with slope  $0$ , containing the point  $(4,2)$

4. parallel to  $2x - 3y = 7$  and passing through  $(5,1)$

5. perpendicular to the line in problem #1, containing the point  $(3,4)$

X. Determine the exact value of each.

1.  $\sin 0$     2.  $\sin \frac{\pi}{2}$     3.  $\sin \frac{3\pi}{4}$     4.  $\cos \pi$     5.  $\cos \frac{7\pi}{6}$     6.  $\cos \frac{\pi}{3}$

7.  $\tan \frac{7\pi}{4}$     8.  $\tan \frac{\pi}{6}$     9.  $\tan \frac{2\pi}{3}$     10.  $\tan \frac{\pi}{2}$     11.  $\cos \left( \sin^{-1} \frac{1}{2} \right)$     12.  $\sin^{-1} \left( \sin \frac{7\pi}{6} \right)$

XI. Determine the domain and range.

1.  $y = \sqrt{x-4}$     2.  $y = \sqrt{x^2-4}$     3.  $y = \sqrt{4-x^2}$     4.  $y = \sqrt{x^2+4}$

XII. Determine all points of intersection.

1.  $y = x^2 + 3x - 4$  and  $y = 5x + 11$

2.  $y = \cos x$  and  $y = \sin x$  in the first quadrant

XIII. Solve for x, where x is a real number.

1.  $x^2 + 3x - 4 = 14$     2.  $\frac{x^4 - 1}{x^3} = 0$     3.  $(x - 5)^2 = 9$

4.  $2x^2 + 5x = 8$     5.  $(x + 3)(x - 3) > 0$     6.  $x^2 - 2x - 15 \leq 0$

7.  $12x^2 = 3x$

8.  $\sin 2x = \sin x, 0 \leq x \leq 2\pi$

9.  $|x-3| < 7$

10.  $(x+1)^2(x-2) + (x+1)(x-2)^2 = 0$

11.  $27^{2x} = 9^{x-3}$

12.  $\log x + \log(x-3) = 1$

13.  $e^{3x} = 5$

XIV. Graph each. State the domain and range.

1.  $y = \sin x$

2.  $y = \cos x$

3.  $y = \tan x$

4.  $y = x^3 - 2x^2 - 3x$

5.  $y = x^2 - 6x + 1$

6.  $y = \frac{x+4}{x-1}$

7.  $y = \frac{x^2 - 4}{x + 2}$

8.  $y = e^x$

9.  $y = \sqrt{x}$

$$10. y = \sqrt[3]{x}$$

$$11. y = \ln x$$

$$12. y = |x+3| - 2$$

$$13. y = \frac{1}{x}$$

$$14. y = \begin{cases} x^2, & \text{if } x < 0 \\ x+2, & \text{if } 0 \leq x \leq 3 \\ 4, & \text{if } x > 3 \end{cases}$$

## ANSWER KEY

### SECTION I:

$$1. \frac{1}{X+1} \quad 2. x^2 + 2X + 4 \quad 3. \frac{-1}{X+5} \quad 4. \frac{X-8}{X-4}$$

### SECTION II:

$$\begin{array}{lll} 1. \sin^2 x + \cos^2 x = 1 & \sec^2 x = 1 + \tan^2 x & \csc^2 x = 1 + \cot^2 x \\ 2. \cos^2 x - \sin^2 x & 2\cos^2 x - 1 & 1 - 2\sin^2 x \\ 3. 2\sin x \cos x & & \end{array}$$

### SECTION III:

$$1. \frac{-h}{x(x+h)} \quad 2. \frac{x^3}{5} \quad 3. \frac{-1}{3(x+3)} \quad 4. \frac{x^2+15}{(x-3)^2(x-1)}$$

### SECTION IV:

$$1. z = \frac{-2x}{5y} \quad 2. z = \frac{4x-y^2}{3y-8}$$

### SECTION V:

$$\begin{array}{llll} 1. 13 & 2. 30 - \sqrt{2} & 3. 4 & 4. \sqrt{51} \quad 5. f^{-1} = \{(5,3), (4,2), (7,1)\} \\ 6. k^{-1} = \sqrt{x-5}, x \geq 5 & 7. \frac{1}{f(x)} = \left\{ \left(3, \frac{1}{5}\right), \left(2, \frac{1}{4}\right), \left(1, \frac{1}{7}\right) \right\} & 8. (kg)(x) = k(x) \cdot g(x) = (x^2+5)\sqrt{x-3} \end{array}$$

### SECTION VI:

$$1. 2x + h - 2 \quad 2. x^3 + 3x^2y + 3xy^2 + y^3 \quad 3. x^{\frac{5}{2}} + x^4 - x^{\frac{7}{2}}$$

### SECTION VII:

$$1. 15 \quad 2. \frac{251}{216}$$

### SECTION VIII:

$$1. \frac{1}{\sqrt{x}} \text{ (SIMPLIFY MEANS WRITE ANOTHER WAY)} \quad 2. 3 \quad 3. \text{ex} \quad 4. 0 \quad 5. 7 \quad 6. -1 \quad 7. -3 \quad 8. -\ln 2$$

$$9. x^3 \quad 10. \frac{x^{\frac{4}{3}}y^3}{3} \quad 11. 9 \quad 12. 20a^{\frac{13}{6}} \quad 13. 8a^{\frac{5}{2}} \quad 14. \frac{3(n+1)}{5}$$

### SECTION IX:

$$\begin{array}{lll} 1. y - 4 = -2(x - 3) & 2. y + 3 = -\frac{5}{6}(x - 1) \text{ or } y - 2 = -\frac{5}{6}(x + 5) & 3. y = 2 \\ 4. y - 1 = \frac{2}{3}(x - 5) & 5. y - 4 = \frac{1}{2}(x - 3) & \end{array}$$

SECTION X:

1. 0      2. 1      3.  $\frac{\sqrt{2}}{2}$       4. -1      5.  $\frac{-\sqrt{3}}{2}$       6.  $\frac{1}{2}$       7. -1  
 8.  $\frac{\sqrt{3}}{3}$       9.  $-\sqrt{3}$       10. UNDEFINED      11.  $\frac{\sqrt{3}}{2}$       12.  $\frac{-\pi}{6}$

SECTION XI:

1.  $domain = [4, \infty)$        $range = [0, \infty)$       2.  $d = [2, \infty) \cup (-\infty, -2]$        $r = [0, \infty)$   
 3.  $d = [-2, 2]$        $r = [0, 2]$       4.  $d = (-\infty, \infty)$        $r = [2, \infty)$

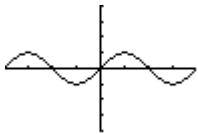
SECTION XII:

1. (5, 36)      (-3, -4)      2.  $\left(\frac{\pi}{4}, \frac{\sqrt{2}}{2}\right)$

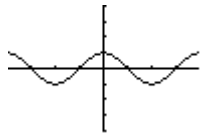
SECTION XIII:

1. -6, 3      2.  $\pm 1$       3. 8, 2      4.  $\frac{-5 \pm \sqrt{89}}{4}$       5.  $(-\infty, -3) \cup (3, \infty)$       6. [-3, 5]      7. 0,  $\frac{1}{4}$   
 8.  $0, \pi, 2\pi, \frac{\pi}{3}, \frac{5\pi}{3}$       9. (-4, 10)      10. -1,  $\frac{1}{2}, 2$       11.  $\frac{-3}{2}$       12. 5 only!      13.  $\frac{\ln 5}{3}$

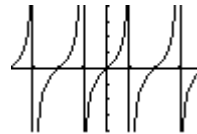
SECTION XIV:



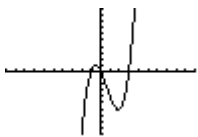
1. D:  $(-\infty, \infty)$   
 R:  $[-1, 1]$



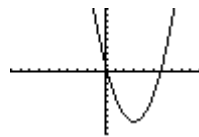
2. D:  $(-\infty, \infty)$   
 R:  $[-1, 1]$



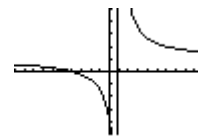
3. D:  $\left\{x : x \neq \frac{(2k+1)\pi}{2}\right\}$   
 R:  $(-\infty, \infty)$



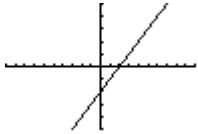
4. D:  $(-\infty, \infty)$   
 R:  $(-\infty, \infty)$



5. D:  $(-\infty, \infty)$   
 R:  $[-8, \infty)$



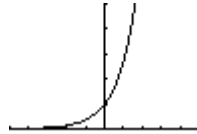
6. D:  $\{x : x \neq 1\}$   
 R:  $\{y : y \neq 1\}$



hole @ (-2,-4)

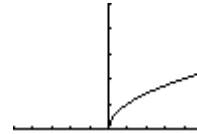
7. D:  $\{x : x \neq -2\}$

R:  $\{y : y \neq -4\}$



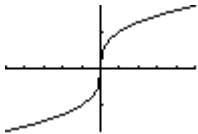
8. D:  $(-\infty, \infty)$

R:  $(0, \infty)$



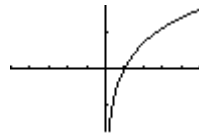
9. D:  $[0, \infty)$

R:  $[0, \infty)$



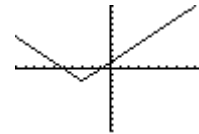
10. D:  $(-\infty, \infty)$

R:  $(-\infty, \infty)$



11. D:  $(0, \infty)$

R:  $(-\infty, \infty)$



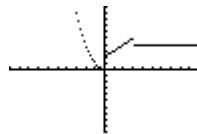
12. D:  $(-\infty, \infty)$

R:  $[2, \infty)$



13. D:  $\{x : x \neq 0\}$

R:  $\{y : y \neq 0\}$



14. D:  $(-\infty, \infty)$

R:  $(0, \infty)$